A CFD based fast method for wind farm layout and energy calculations

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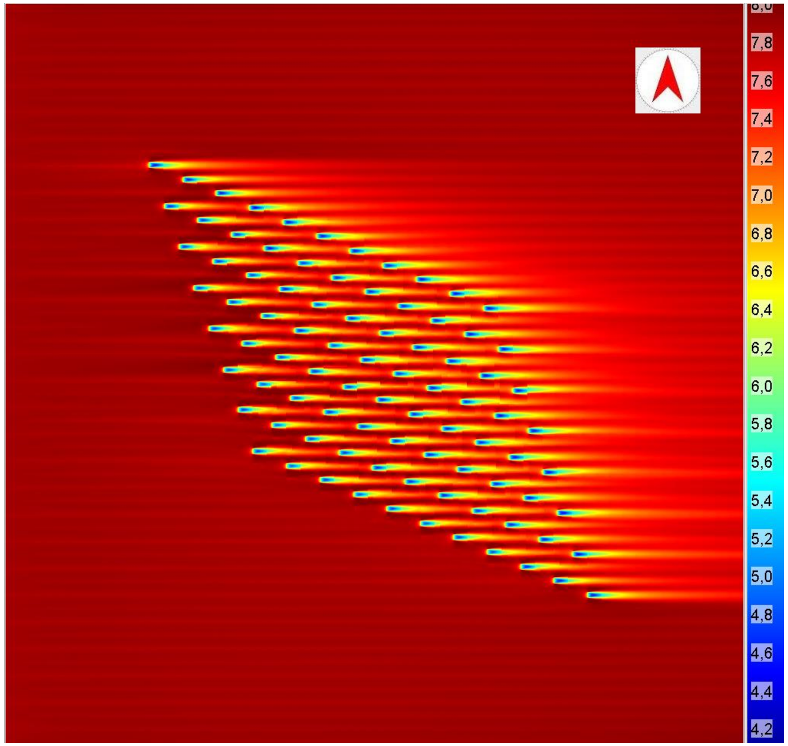
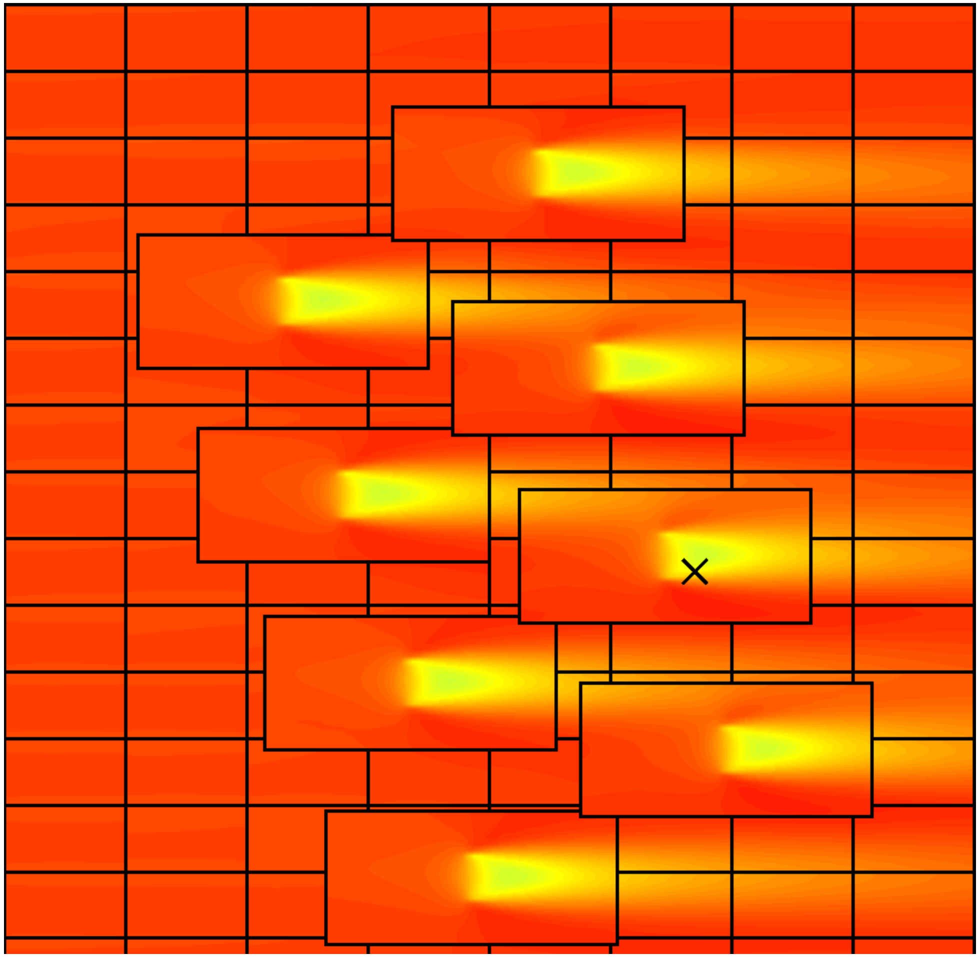
About our implementation of model reduction

- Reduction of the solution space, while including the nonlinear RANS equations
- We divide the wind farm into 3-dimensional tiles, so that turbines can be placed in arbitrary positions
- The solution space is constructed from a set of steady state CFD simulations using the RANS equations
- The solution time for new solutions is significantly reduced compared to CFD



Empty tiles and the background grid



- We have added "empty tiles" to propagate the flow field between non-overlapping turbine tiles
- The physical dimension of empty tiles is smaller than turbine tiles

This years objectives

- Validation of results against measurement data at Sheringham Shoal
  - Construct basis for the Sheringham Shoal turbine
- Speed and memory usage improvements

Construction of solution space for the Sheringham Shoal turbine

	Previous years	Sheringham Shoal
Rated power	2 MW	3.6 MW
Rotor diameter	76 m	107 m
Hub height	64 m	81.75 m

- The turbine tiles are constructed as horizontally <u>square</u> tiles with dimensions 5 rotor diameter width and length
  - The height is approx. 1.3 km

Experienced technical challenges

- Could not represent large (sparse) matrices in the linear algebra library
  - Solved by making our own implementation of sparse block matrices!

- The solution time for the setup of Sheringham Shoal took several minutes on a 4 core desktop PC
  - Improved to less than 1 minute by making speed optimizations of the solution algorithm

Solving the reduced order equations

- Minimize the following objective function:

$$2F(x) = (1-\alpha)\|B(x)\|^2 + \alpha\|T(x)\|^2$$

Tile boundary coupling → $B(x)$ RANS equations → $T(x)$

- Using the Newton-Raphson method:

$$x_{k+1} = x_k - H(F(x_k))^{-1}\,\nabla F(x_k)$$

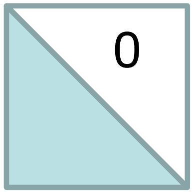
$$A(x_k)(x_{k+1} - x_k) = -\nabla F(x_k)$$

Solving the reduced order equations (cont.)

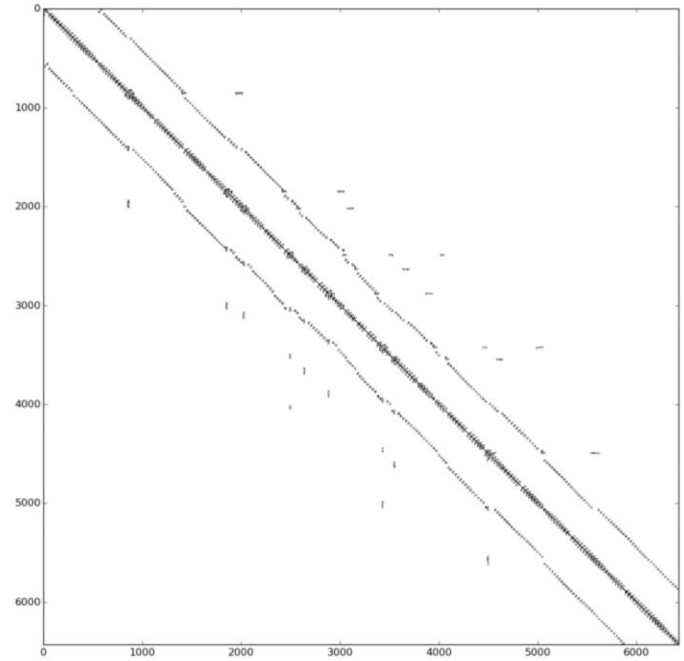
$$Ap = b$$

- A is a symmetric, positive definite and sparse block matrix
- Initially, we used an exact solver: The Cholesky-decomposition:

$$A = LL^T$$

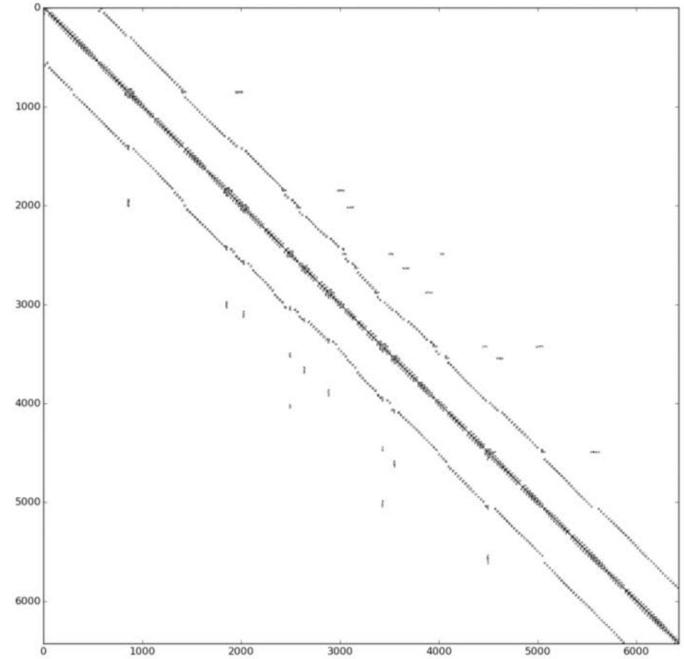
$L \sim$ 

- Challenge: Even though A is sparse, L is not necessarily sparse
  - Can be "solved" by finding a permutation matrix P, s.t. $P^T AP = LL^T$ with L sparse – the library Eigen has a module which does this



Sparsity of the matrix

- Can show that the fraction of non-zeros for a grid of empty tiles with $nx \cdot ny$ tiles is $\frac{5}{nx \cdot ny}$



- For the Sheringham Shoal setup, the fraction of non-zeros is 0.16%

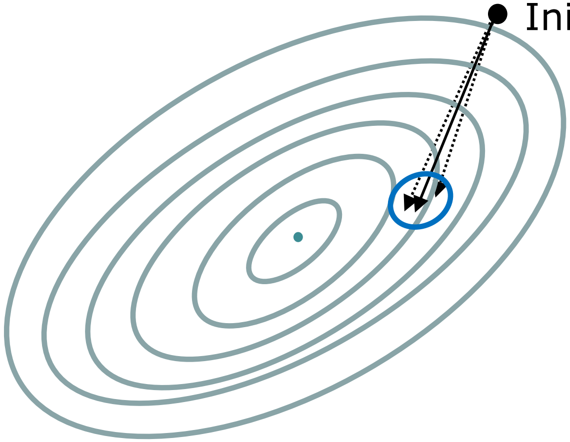
Solving with the conjugate gradient method

- Features
  - Iterative method
  - The most computationally intensive operation in each iteration is a matrix-vector product
- We are using the block diagonal matrix elements of A as preconditioner

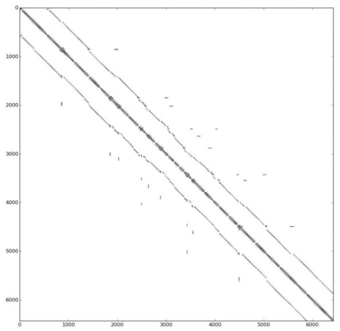
- Benefits over Cholesky:
  - Less memory intensive
  - Faster on very large systems

Exploitable problem features to improve the solution speed

- Terminating CG iterations early improves the computational effort



- The matrix-vector calculations can be parallelized, due to the sparsity of the matrix A
  - Conventional wisdom says that it does not help to parallelize a matrix-vector calculation because the problem is memory bandwidth bound



Speed optimizations

- Our setup of Sheringham Shoal consists of 3025 'empty tiles' and 88 turbine tiles
- This covers the wind farm domain by 112 million grid cells, which corresponds in our reduced space to ~100 000 degrees of freedom

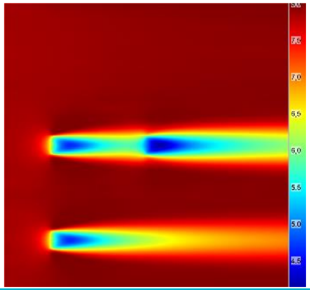
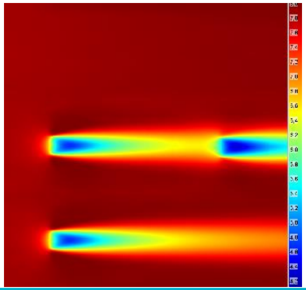
Matrix system solution speeds for a 4 core desktop PC:

Cholesky with permutations	Conjugate gradient, no preconditioner	Conjugate gradient, with block diagonal preconditioner
~7 min	~6 min	~25 s

Verification studies against CFD

- More details in previous presentations

Case	Max deviation of single turbine power	Deviation of total power
10 turbines in a row	3 %	1 %
Pyramid setup, downstream distance 5D	7 %	< 2 %
Pyramid setup, downstream distance 9D	5 %	< 1.5 %

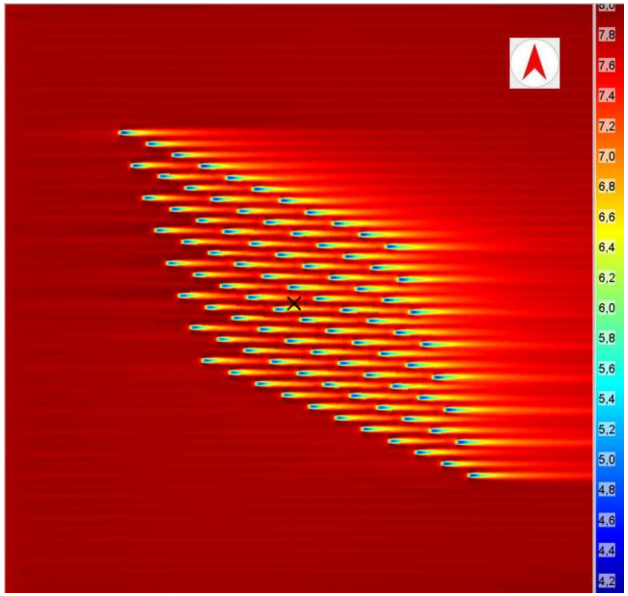



Setting up the Sheringham Shoal simulations

- Obtained the locations of all the turbines (A1 – K8)

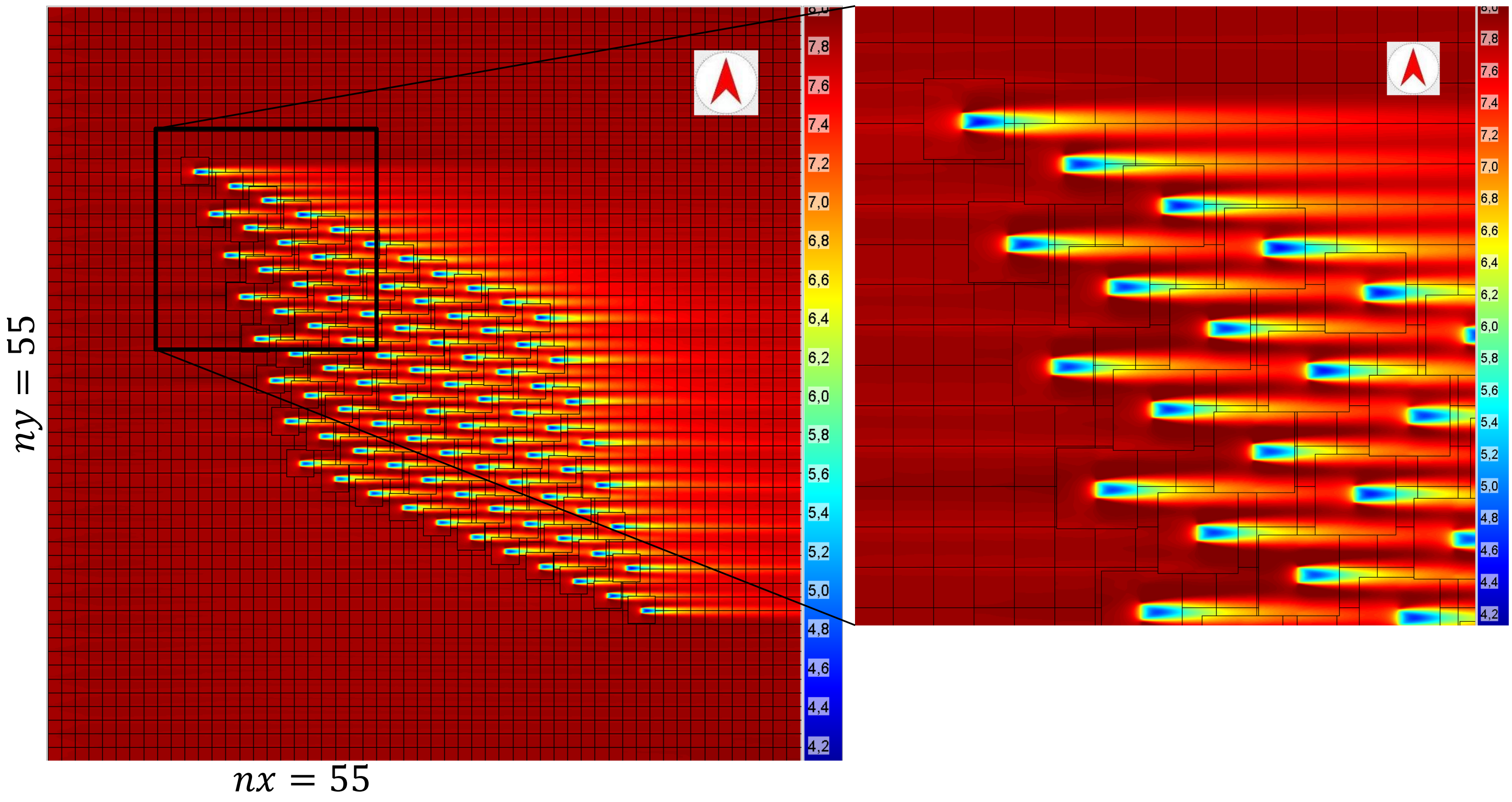
- Inflow wind speed from a neutrally stratified profile with around 8 m/s at hub height

(7476.5m, 7476.5m)

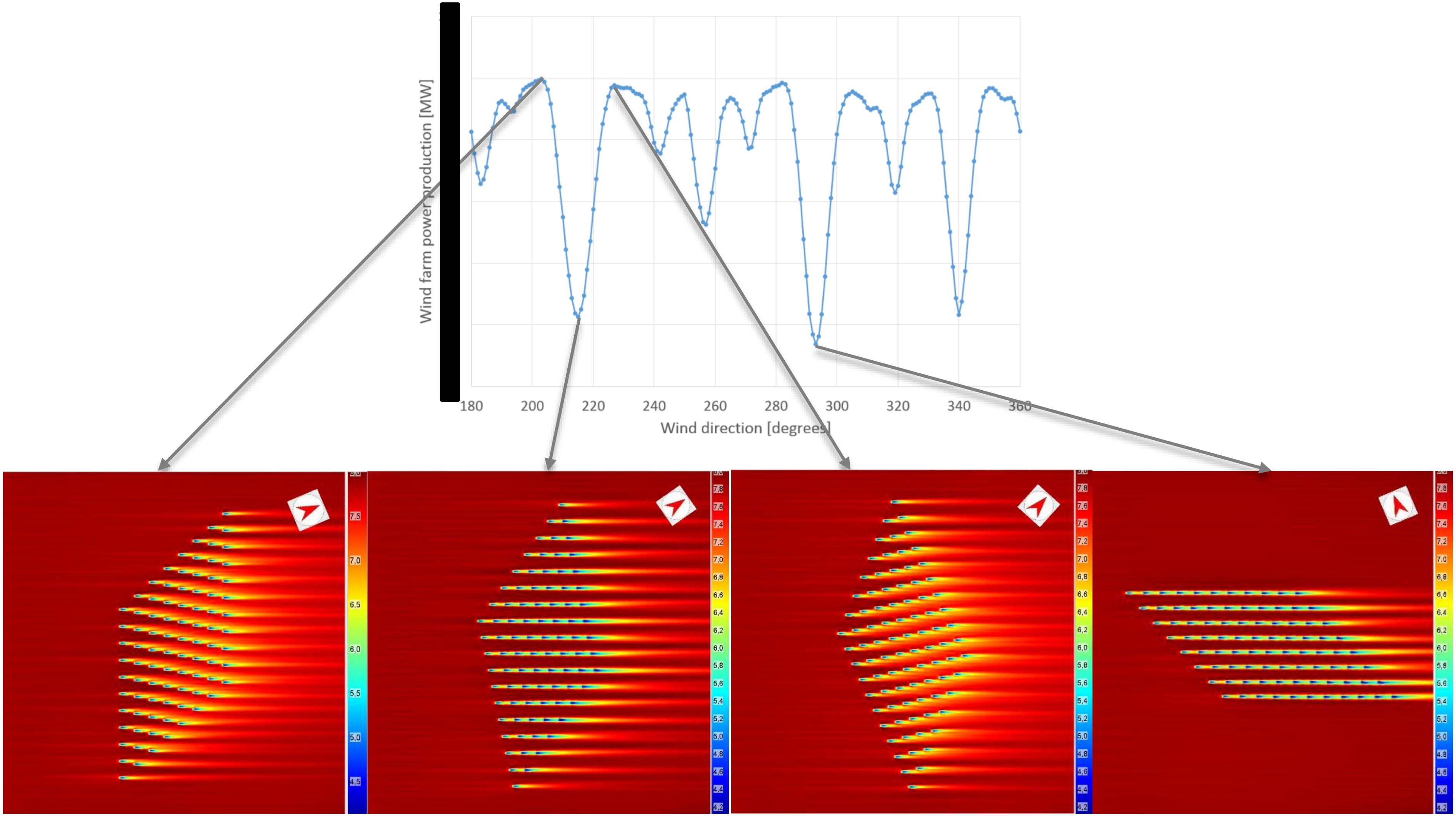


(-7236m, -7236m)

The tile grid for Sheringham Shoal



Example simulations



Discussion / further work

- Speed optimizations by a factor of 20 (from minutes to seconds)
- We are able to simulate a Sheringham Shoal setup, for multiple wind directions
- Validation results to be presented at internal workshop on Sheringham Shoal data for NORCOWE members Nov 2nd